Gig Economy: A Dynamic Principal-Agent Model*

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Abstract

The gig economy, where employees take short-term, project-based jobs, is increasingly spreading all over the world. In this paper, we investigate the employer's and the worker's behavior in the gig economy with a dynamic principal-agent model. In our proposed model the worker's previous decisions influence his later decisions through his dynamically changing participation constraint. He accepts the contract offered by the employer when his expected utility is higher than the irrational valuation of his effort's worth. This reference point is based on wages he achieved in previous rounds. We formulate the employer's stochastic control problem and derive the solution in the deterministic limit. We obtain the feasible net wage of the worker, and the profit of the employer. Workers who can afford to go unemployed and need not take a gig at all costs will realize high net wages. Conversely, far-sighted employers who can afford to stall production will obtain high profits.

Keywords: dynamic principal-agent problem, gig economy
JEL: C73, D82, D86, J33, J41

1 Introduction

In the gig economy, companies and workers prefer the flexible, fixed-term, part-time employment over the long-term, full-time employment. Employees in the gig economy take short-term, project-based jobs, so-called gigs. By doing so, they flexibly adapt to their own needs, as well as the needs of the companies and their customers. The gig economy is increasingly spreading, inter alia, in the fields of art and design (e.g. musicians, graphics designers, fine and applied artists etc.), in information technology and computer science (e.g. web and software developers, programmers, cybersecurity experts etc.), in constructions and productions (e.g. carpenters, engineers etc.), in media and communications (e.g. journalists, photographers, translators etc.) and in transportation (eg. cab drivers, truck drivers etc.). This phenomenon accelerated after the 2008 financial crisis, as millennials who start working nowadays have found that jobs that were previously safe and meant long-term careers are not as certain as they previously seemed to be. For them, the personal freedom that they can decide when, what and how much they want to work is appreciated more than the steady income. Part of this freedom is that freelancers and atypical workers have more control over their work-life balance than traditional workers. Short-term, project-oriented self-employment, freelance entrepreneurship, and the related gig economy are increasingly spreading all

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over the world. It is enough to look at the headlines of some of the most prestigious journals such as The New York Times or Forbes: "Freelancers in the 'Gig Economy' Find a Mix of Freedom and Uncertainty" (Swarns [2014]), "The Rise Of The Freelancer Economy" (Rashid [2016]).

Academic research on gig economy and freelance employment is just beginning. Katz and Krueger [2016] studies atypical employment in the United States between 1995 and 2015, Friedman [2014] and Torpey and Hogan [2016] analyze the macroeconomic and socio-political aspects of the gig economy, Donovan et al. [2016] gives an overview of the gig economy in the U.S., Kuhn [2016] approaches the issues raised by the gig economy from the industrial and organizational psychology side, Burtch et al. [2018] examines the impact of the gig economy on traditional entrepreneurial activity.

In the economy, especially in the gig economy, the employers hire the workers to act on behalf of them during their work. This asymmetric relationship is theorized by agency theory of Jensen and Meckling [1976]. "We define an agency relationship as a contract under which one or more persons (the principal(s)) engage another person (the agent) to perform some service on their behalf which involves delegating some decision making authority to the agent." (Jensen and Meckling [1976][p. 308]) The decision-making right is transferred from the principal to the agent, but the agent does not necessarily act on the same interests as the principal on whose behalf he acts. Because of this conflict of interest, moral hazard can exist in the system. Following Hart and Holmström [1986], we define moral hazard as the risk which comes from the agent's hidden action. Hidden action means that the agent's activity, the effort is unknown to the principal, either because it is simply not observable, cannot be verified or because observing it would be very costly. The model outlined above, and the difficulties that we encounter are commonly referred to as the principal-agent problem.

One of the possible mechanisms that can reduce or even eliminate the moral hazard in the principal-agent problem is a contract between the two parties that is enforceable. With such contract, the principal may incentivize the agent to carry out the production or service activity in a manner to his own interests. The field of contract theory deals with the design and examination of such contracts that are optimal for some aspect. Literature includes Holmström and Milgrom [1991], Holmström [1999], Meza and Webb [2007], Dittmann et al. [2010] and Edmans et al. [2012] in which the authors analyze the incentive effects of managerial contracts. Tirole's book (Tirole [2006]) summarizes and synthesizes corporate finance research in incentive and contract theory frameworks. Bhattacharya and Pfleiderer [1985], Stoughton [1993], Heinkel and Stoughton [1994], Stracca [2006] analyzes the relationship between portfolio managers and investors. Otto [2006] presents various forms of mining royalties. Among others, Lafontaine [1992], Lafontaine and Shaw [1999] and Fan et al. [2017] examine franchise contracts in the principal-agent framework. Sherer et al. [1998], Rawley and Simcoe [2010], Cohen and Kietzmann [2014] investigate the relationship between taxi and community-based ridesharing companies to workers and government from the agency theory point of view. Van Dijck [2009], Ghosh and McAfee [2011], Tang et al. [2012] and Khansa et al. [2015] examine the motivational and incentive issues of user generated contents (UGC) in Web 2.0. Bolton and Dewatripont [2005] provides excellent systematization and summaries for the mathematical models of contract theory.

The short-term and project-based nature of the gig economy means that performance is evaluated and the terms of the contract are modified from project to project, in contrast to the long-term or indefinite-term contracts. Due to these frequently changing contracts, it is necessary to model the gig-economy as a dynamic principal-agent problem. Plambeck and Zenios [2000], Zhang and Zenios [2008] model dynamic principal-agent problems with Markov decision processes (MDP). Sannikov [2008] examines the principal-agent problem and long-term contracts in a continuous stochastic model. Edmans et al. [2012] applies this to examine the appropriate incentive system for corporate CEOs, DeMarzo and Sannikov [2016] and He et al. [2017] continue to develop the model with learning and information rent. Cvitanic et al. [2018] introduce a general methodology to reduce the principal-agent stochastic control problem to a stochastic game. These contributions introduce dynamics to the principal-agent problem by dynamics of the output. When representing the principal-agent problem as a dynamic game (or as a decision process), the influence of iterative contracts to the agent has not been considered. In this paper, our goal is to build a dynamic model in which previous contracts and the payments influence not only the principal's but also the agent's decisions.

The ever-changing contractual environment can unscramble the agent and may lead to him behaving irrationally. Acknowledging and modeling irrational behavior as the reality of human decision making goes back to the prospect theory of Kahneman and Tversky (Kahneman and Tversky [2013],Tversky and Kahneman [1992]). A more recent example that motivated our own approach is the reference-based utility model of Kőszegi and Rabin (Kőszegi and Rabin [2006], Kőszegi and Rabin [2007]).

In our proposed model, the agent is risk-averse and myopic, optimizing his effort to achieve maximum utility only for the current round. Irrationality of the agent is captured by him only accepting the linear contract (defined by two parameters, a variable component, typically a share which is the linear function of the output, and a fixed component) if its expected utility is higher than his current reference point, even though rejecting the offer may result in a lower utility for him. This behavior represents an irrational valuation of his effort's worth, based on an exponential weighted moving average achieved in previous rounds. In words more familiar in the moral hazard literature, the previous decisions of the players influence later decisions through the agent's dynamically changing participation constraint. In contrast to the agent's risk-averse and myopic behavior, the principal is risk-neutral and plans for the long term, optimizing on an infinite time horizon. Our model, as in most of the cited research, leads to a stochastic control problem. The principal must specify the parameters of the linear contract, depending on the agent's current reference point.

In Section 2 we introduce our proposed dynamic principal-agent model and formulate the employer's stochastic control problem. In Section 3 we derive the solution of this control problem in the deterministic limit. We conclude that workers who can afford to go unemployed and need not take a gig at all costs will realize high net wages. Conversely, far-sighted employers who can afford to stall production will obtain high profits.

2 Model setup

We formulate our proposed dynamic principal-agent model keeping the gig economy context in mind, therefore throughout the paper, we will often refer to the principal as the employer (she), and to the agent as the worker (he). The output of production is $x_t := \chi_t \cdot (z_t + \varepsilon_t)$, where z_t is the effort level of the worker, and $\varepsilon_t \sim N(0, \sigma^2)$ is a normally distributed random noise term representing an uncertainty in the result. $t \in \mathbb{Z}^+$ denotes the time index (*round*) in the model. χ_t is the contract indicator ($\chi_t = 1$ if a contract is struck between the employer and the worker in the current round, and $\chi_t = 0$ if not). It is the worker who decides whether he accepts the contract and works or not. However, we assume the employer is perfectly aware of the preferences of the worker, as well as of his current reference level. Therefore the employer, by offering a "reasonable" or "unreasonable" contract, can, in essence, decide whether the worker will work or not. From now on, we will treat the contract indicator as a control variable at the employer's disposal.

The employer offers a linear contract (*wage*) to the worker:

$$w_t := \chi_t \cdot (s_t \cdot \underbrace{(z_t + \varepsilon_t)}_{x_t} + f_t), \tag{1}$$

where $s_t \in \mathbb{R}$ is the fraction of the output offered to the worker. This determines the variable (*share*) component, while $f_t \in \mathbb{R}$ is the *fix* component of the worker's wage. This fix amount may be negative, in that case it is interpreted as a rent. The remainder part of the output, after subtracting the fix wage, determines the employer's *profit*:

$$\pi_t := \chi_t \cdot ((1 - s_t) \cdot (z_t + \varepsilon_t) - f_t) \tag{2}$$

The employer is risk neutral. The worker is risk averse with a constant absolute risk-averse (CARA) risk preference, represented by a negative exponential *utility function*:

$$u(v_t) := -\chi_t \cdot \exp\left\{ -\gamma \cdot \left(w_t - c \cdot \frac{z_t^2}{2} \right) \right\}, \tag{3}$$

where γ is the worker's *coefficient of absolute risk aversion* ($\gamma > 0$) and the *effort cost* ($c \cdot \frac{z_t^2}{2}$) is measured in the same monetary units as the wage ($c \in \mathbb{R}^+$). The *net wage* of the worker is the wage minus the effort cost. Substituting, we obtain

$$v_t := \chi_t \cdot \left(\underbrace{s_t \cdot (z_t + \varepsilon_t) + f_t}_{w_t} - c \cdot \frac{z_t^2}{2} \right) \tag{4}$$

It is straight forward to calculate the worker's *expected utility*, we obtain

$$E\big[u(v_t)\big] = -\chi_t \cdot \exp\left\{ -\gamma \cdot \left(s_t \cdot z_t + f_t - c \cdot \frac{z_t^2}{2} - \frac{\gamma \cdot s_t^2 \cdot \sigma^2}{2} \right) \right\} \tag{5}$$

The worker is myopic, he wants to maximize this expected utility w.r.t. his effort z_t in each round, this yields

$$z_t^* = \chi_t \cdot \frac{s_t}{c} \tag{6}$$

The worker takes the job if the expected utility of the offer in the new round is greater than her current *reference utility* which we model as the utility of a deterministic *reference value* (cash equivalent).

$$E\big[u(v_t)\big] \geq u(R_t) \tag{7}$$

This (ineq. 7) is the participation constraint of the worker. We choose to model R_t as the exponentially-weighted running average of the previous realized net wages. So the reference is updated as:

$$R_{t+1} := \beta \cdot R_t + (1-\beta) \cdot \underbrace{\chi_t \cdot \left(s_t \cdot (z_t + \varepsilon_t) + f_t - c \cdot \frac{z_t^2}{2} \right)}_{v_t}, \tag{8}$$

where $\beta \in (0,1)$ is the *decay parameter* of the worker's memory. The greater β is (the closer it is to 1), the more rounds the worker looks back to previous net wages for establishing his reference level.

Now let us think about the employer's decision dilemma. She wants to maximize the expected value of her *lifetime profit*

$$\Pi_t = (1-\delta) \cdot \sum_{i=t}^{\infty} \delta^{i-t} \cdot E[\pi_i], \tag{9}$$

where $\delta \in (0,1)$ is her *subjective discount factor*. The greater δ is, the more the employer values profits realized in the long term, relative to the instant profit realized in the current round. We introduced the prefactor $(1-\delta)$ in the definition so that Π_t is actually the lifetime average profit per round. The employer knows all of the parameters which characterize the worker (γ, c, β) and the stochastic environment ($\varepsilon_t \sim N(0, \sigma^2)$). So the employer's optimization problem is the following:

$$\max_{\{\chi_i, s_i, f_i\}_{i \geq t}} (1-\delta) \cdot \sum_{i=t}^{\infty} \delta^{i-t} \cdot E[\pi_i], \; \forall t \tag{10}$$

s.t.

$$z_t = \operatorname*{argmax}_{\hat{z}_t} E\left(u\left(v_t\right)\right) \tag{ICC}$$

$$E\left(u\left(v_t\right)\right) \geq u(R_t) \tag{PC}$$

$$R_{t+1} = \beta \cdot R_t + (1-\beta) \cdot v_t \tag{RU}$$

In each round t the employer maximizes her remaining lifetime profit (Π_t). She decides whether she wants the worker to work (χ_t), and if she does, then she decides on the share (s_t) and on the fix wage (f_t). The worker is risk-averse and myopic. He chooses his effort level to maximize his expected utility in the given round (ICC), and if this maximal expected utility is not less than the utility of his reference point, he takes the offer (PC). The reference point of the worker is the exponentially-weighted running average of previous realized net wages (RU).

If the worker does not get the job, then the worker realizes a wage $w_t = 0$, net wage $v_t = 0$, and the employer realizes a profit $\pi_t = 0$. If the worker gets the job, his effort will be $z_t = \frac{s_t}{c}$. In this case, depending on the new random ε_t, the worker will realize the net wage

$$v_t = w_t - c \cdot \frac{z_t^2}{2} = s_t^2 \cdot \frac{1}{2 \cdot c} + s_t \cdot \varepsilon_t + f_t \tag{11}$$

and the employer will realize the profit

$$\pi_t = (1-s_t) \cdot x_t - f_t = (1-s_t) \cdot (z_t + \varepsilon_t) - f_t = (1-s_t) \cdot \left(\frac{s_t}{c} + \varepsilon_t \right) - f_t \tag{12}$$

The expected value of the net wage and the profit, seen before the realization of the random ε_t:

$$E[v_t] = s_t^2 \cdot \frac{1}{2 \cdot c} + f_t \tag{13}$$

$$E[\pi_t] = (1 - s_t) \cdot \frac{s_t}{c} - f_t \tag{14}$$

It is intuitively obvious, that even in the multi-period game, if the employer wants to maximize his expected lifetime profit and also wants the worker to work ($\chi_t = 1$), she will set f_t as low as possible. We will not prove this rigorously here, but treat it as an assumption and only offer some (hopefully) convincing arguments: In the current round, a higher fix wage would not incentivize the worker more, but it would lower the employer's profit. A higher fix wage would increase the worker's current wage, thereby his reference level for future rounds, which is also disadvantegeous for the employer.

This means, that optimization of f_t is trivial (ineq. 7 is binding), so in this case ($\chi_t = 1$) we have

$$f_t^* = R_t - s_t^2 \cdot \frac{1}{2 \cdot c} + \frac{\gamma \cdot s_t^2 \cdot \sigma^2}{2} \tag{15}$$

Plugging it into the employer's expected profit, we obtain

$$E[\pi_t] = (1 - s_t) \cdot \frac{s_t}{c} - R_t + s_t^2 \cdot \frac{1}{2 \cdot c} - \frac{\gamma \cdot s_t^2 \cdot \sigma^2}{2} \tag{16}$$

If the employer wants to optimize $E[\pi]$ (this is the one-shot game), then we would get the familiar result for the optimal s_t^* as

$$s_t^* = \frac{1}{1 + c \cdot \gamma \cdot \sigma^2} \tag{17}$$

But now we are looking at the multi-period game, so this is not necessarily true. Putting things together, this is how the multi-step control problem looks like from the employer's point of view at round t:

1. Observe reference of worker R_t.
2. Decide on whether she wants to give a reasonable contract so the worker will work χ_t. *Intuitively it is obviously* a threshold-decision (she offers a reasonable contract if R_t is smaller than a threshold $\bar{R}$). We do not prove it for now, we make it an assumption.

$$\chi_t := \begin{cases} 1, & R_t \leq \bar{R} \\ 0, & R_t > \bar{R} \end{cases} \tag{18}$$

3. (a) If no contract is struck ($\chi_t = 0$), then the employer obtains $E[\pi_t] = 0$, worker's reference is updated as $R_{t+1} = \beta \cdot R_t$.

3. (b) If contract is struck ($\chi_t = 1$), then employer has to decide on s_t. This decision can only depend on R_t, so the control is a function $s_t(R_t)$.

4. (b) $f_t^* = R_t - s_t^2 \cdot \frac{1}{2 \cdot c} + \frac{\gamma \cdot s_t^2 \cdot \sigma^2}{2}$ is trivially decided once $s_t(R_t)$ is decided, so this does not add dimensionality to the control problem.

5. (b) The worker decide his optimal effort level $z_t^* = \frac{s_t}{c}$.

6. (b) The worker checks his participation constraint, and if the expected utility is not less than his reference utility he will work with effort level z_t.

$$E[\pi_t] = \frac{s_t}{c} - s_t^2 \cdot \left(\frac{1}{2 \cdot c} + \frac{\gamma \cdot \sigma^2}{2}\right) - R_t \tag{19}$$

(Actually, as the employer foresaw the worker's decision process, and decided to give him a reasonable contract, the worker will work.)

7. (b) The ε_t stochastic component is realized and the output is $x_t = z_t^* + \varepsilon_t$.

8. (b) The employer obtains the profit

$$\pi_t = \frac{s_t}{c} + (1 - s_t) \cdot \varepsilon_t - s_t^2 \cdot \left(\frac{1}{2 \cdot c} + \frac{\gamma \cdot \sigma^2}{2}\right) - R_t \tag{20}$$

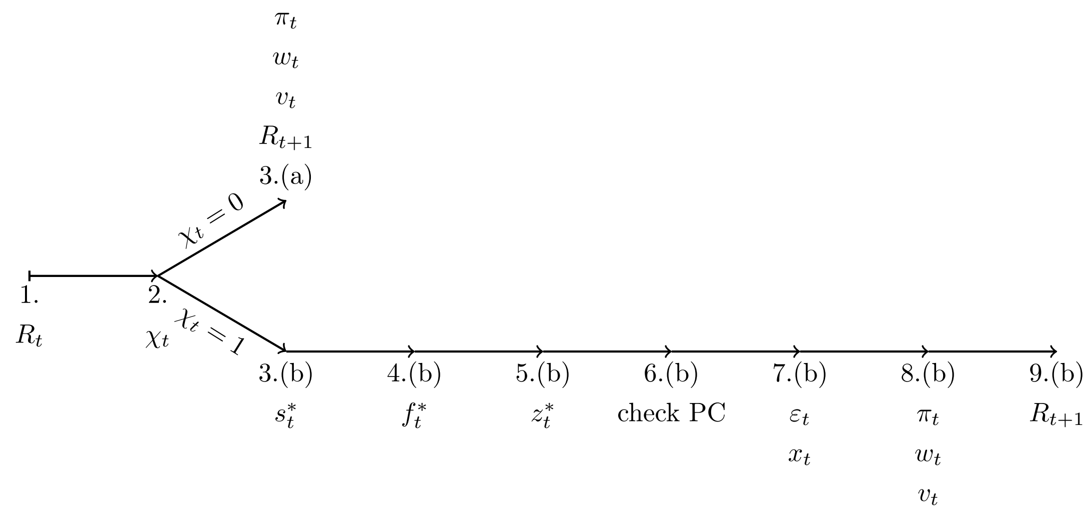

Figure 1: Figure shows the process of decisions and determinations of the different variables at round t.

The worker receives his wage

$$w_t = s_t \cdot \varepsilon_t + s_t^2 \cdot \left(\frac{1}{2 \cdot c} + \frac{\gamma \cdot \sigma^2}{2} \right) + R_t \tag{21}$$

Accordingly, the worker's net wage is

$$v_t = s_t \cdot \varepsilon_t + \frac{\gamma \cdot s_t^2 \cdot \sigma^2}{2} + R_t \tag{22}$$

9. (b) Worker's reference in this case is updated as

$$R_{t+1} = \beta \cdot R_t + (1 - \beta) \cdot v_t = R_t + (1 - \beta) \cdot \left(s_t \cdot \varepsilon_t + \frac{\gamma \cdot s_t^2 \cdot \sigma^2}{2} \right) \tag{23}$$

See a schematic representation of the decision process in Figure 2.

3 Solution of the deterministic case

In this section we solve the optimal strategy of the employer when the environment is not stochastic, so $\sigma = 0$. Eliminating the noise from the output has two simplifying consequences in the problem: First, the worker's risk-awareness does not matter, as he does not face uncertainty in the output. Second, the dynamics of the reference value will lose its stochastic component. This makes the control problem also deterministic, and it is simplified as

- No contract struck ($\chi_t = 0$)

$$E[\pi_t] = 0 \tag{24}$$
$$R_{t+1} = \beta \cdot R_t \tag{25}$$

- Contract struck ($\chi_t = 1$)

$$E[\pi_t] = \frac{s_t}{c} - s_t^2 \cdot \frac{1}{2 \cdot c} - R_t \tag{26}$$
$$R_{t+1} = R_t \tag{27}$$

What makes this optimal control problem tractable is the fact that the dynamics of R_t does not depend on the s_t control variable, only on the contract/no contract decision (χ_t). So the employer will always locally optimize s_t, as it has no future consequences. This leads to the trivial $s_t = 1$ result, whenever contract is sought. So now we are left with the optimal control problem with only the contract/no contract decision:

- No contract struck ($\chi_t = 0$)

$$E[\pi_t] = 0 \tag{28}$$

$$R_{t+1} = \beta \cdot R_t \tag{29}$$

- Contract struck ($\chi_t = 1$)

$$E[\pi_t] = \frac{1}{2 \cdot c} - R_t \tag{30}$$

$$R_{t+1} = R_t \tag{31}$$

Again, we will assume (and we believe it can be proven) that the contract/no contract decision is a threshold-decision with threshold $R_t = \bar{R}$ (see eq. 18). This means, that at the threshold, the employer should be indifferent with the contract/no contract choice. The employer wants to maximize her lifetime profit

$$V(R_t) = \max_{\{\chi_i(R_i)\}_{i \geq t}} (1 - \delta) \cdot \sum_{i=t}^{\infty} \delta^{i-t} \cdot E[\pi_i] \tag{32}$$

If the optimal choice is contract ($R_t < \bar{R}$ so $\chi_t = 1$), then the worker's reference does not move, so there will always be contract from then on. So in this case:

$$V_{contr}(R_t) = (1 - \delta) \cdot \sum_{i=t}^{\infty} \delta^{i-t} \cdot E[\pi_i] = (1 - \delta) \cdot \sum_{i=t}^{\infty} \delta^{i-t} \cdot \left(\frac{1}{2 \cdot c} - R_i \right) = \frac{1}{2 \cdot c} - R_t \tag{33}$$

Now assume the worker's reference is just a little above the threshold ($R_t > \bar{R}$). First choice is no contract, that brings the reference down below the threshold, and then perpetual contract happens. So now:

$$V_{nocontr}(R_t) = 0 + V_{contr}(R_{t+1}) = (1 - \delta) \cdot \sum_{i=t+1}^{\infty} \delta^{i-t} \cdot \left(\frac{1}{2 \cdot c} - R_i \right) = \delta \cdot \left(\frac{1}{2 \cdot c} - \beta \cdot R_t \right) \tag{34}$$

At the threshold ($R_t = \bar{R}$), these are equal (eq. 33 and eq. 34), so we obtain

$$\frac{1}{2 \cdot c} - R_t = \delta \cdot \left(\frac{1}{2 \cdot c} - \beta \cdot R_t \right) \tag{35}$$

Solving for $\bar{R}$, we obtain

$$\bar{R} = \frac{1}{2 \cdot c} \cdot \frac{1 - \delta}{1 - \delta \cdot \beta} \tag{36}$$

So what will happen? The employer offers a reasonable contract if and only if the worker's reference level is below the threshold value. In this case, she always offers full share on the output ($s_t = 1$) for a rent (negative f_t). The rent is chosen so the worker will just be willing to work. If the reference of the worker R_0 is initially below the threshold $\bar{R}$, then it remains at the same low level. The worker will work in all consequent rounds. If the reference R_0 is initially above the threshold $\bar{R}$, then the first few rounds go with no contract. In this case, the reference R_t decays exponentially. As soon as it goes below the threshold $\bar{R}$, it remains constant, and the worker will always work from then on. Figure(2) illustrates this dynamics.

Our result shows a disheartening feature of the labor market: Workers that choose a low starting point as their initial reference, tend to get stuck at this low level in terms of their long-term net wage. Although the employer could raise the worker's wage, she is not forced to; in our model, the worker "gets used to" his low wage, and his expectations do not grow. A practical advise the model suggests for entrants, is to represent as high an expectation as possible at the first interview, and in case of failure, lower it only gradually. The worker may skip a few rounds of employment initially, but he would make sure his eventual net wage will be close to the maximally attainable level, that is, to $\bar{R}$. In our analysis that follows we assume the worker makes the optimal choice, starting at the $\bar{R}$ reference level, thereby ensuring a constant net wage at the same $\bar{R}$ level forever. In this case, the employer's profit per round will also be constant at

$$V(\bar{R}) = \frac{1}{2 \cdot c} \cdot \frac{\delta \cdot (1 - \beta)}{1 - \delta \cdot \beta} \tag{37}$$

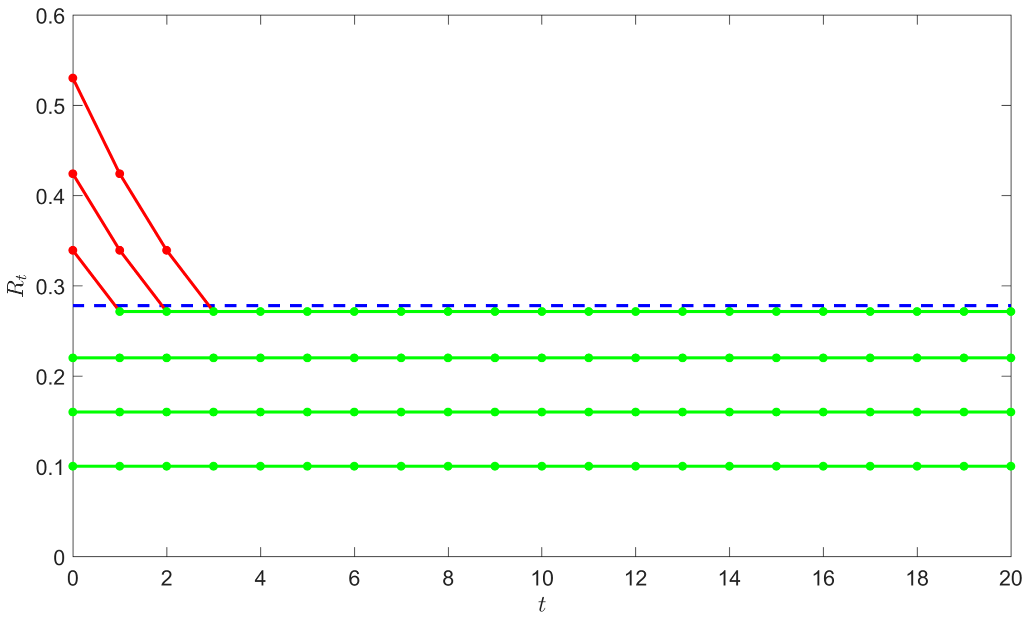

Figure 2: The net wage of the worker (R_t) in the first 20 rounds from different initial reference points ($R_0 \in \{0.53, 0.42, 0.34, 0.22, 0.16, 0.1\}$). The dashed blue line represents the threshold value of the reference point ($\bar{R} = 0.28$) with $\beta = 0.8$, $\delta = 0.8$ and $c = 1$. Red markers show instances when the reference point is greater than the threshold, in these cases no contract is struck ($\chi = 0$). Green markers on the other hand represent situations when the reference point is less than the threshold, in these cases contract is struck ($\chi = 1$).

As now $s_t = 1$ constant, the effort, the cost of effort and the production output will also be constant at $z_t = \frac{1}{c}$, $cost = \frac{1}{2 \cdot c}$ and $x = \frac{1}{c}$, respectively. Subtracting the cost of effort from the output, we obtain the net value of production at a constant $\frac{1}{2 \cdot c}$. The employer and the worker share this net production in the form of profit and net wage. Indeed, simple calculation shows that $\bar{R} + V(\bar{R}) = \frac{1}{2 \cdot c}$. In Figure(3), we plot the worker's optimal net wage ($\bar{R}$) and the corresponding employer profit ($V(\bar{R})$) as functions of the worker's memory parameter (β), at different employer discount factor (δ) values. Figure(3) shows that the worker's optimally attainable net wage increases, while the employer's profit decreases with β. Conversely, the worker's share from the net production decreases and the employer's profit increases with δ.

4 Discussion and Conclusions

In this paper, we introduced a Dynamic Principal-Agent Model to illustrate some of the aspects of wage bargaining in the gig economy. After a general stochastic formulation, we derived formulae for the worker's net wage and the employer's profit in the deterministic limit. Our results are shown in Figure(3). Another way to illustrate how the employer and the worker share the net output, is to observe the ratio

$$\frac{V(\bar{R})}{\bar{R}} = \frac{\delta \cdot (1 - \beta)}{1 - \delta}. \tag{38}$$

Here we offer some discussion on this result. A small β value means the worker is very sensitive to missing a round of employment; he lowers his reference point significantly in that case. Our model suggests such workers are at a disadvantage in terms of their wage. On the other hand, employees who can go without work for several rounds without seriously lowering their standards will end up with higher wages. Reasons for the latter may be that the worker has savings, or he has a marketable skill, so he can afford to wait for realistic offers, rather than taking jobs with subpar compensation. From the employer's perspective, a high δ means she appreciates long-term profit

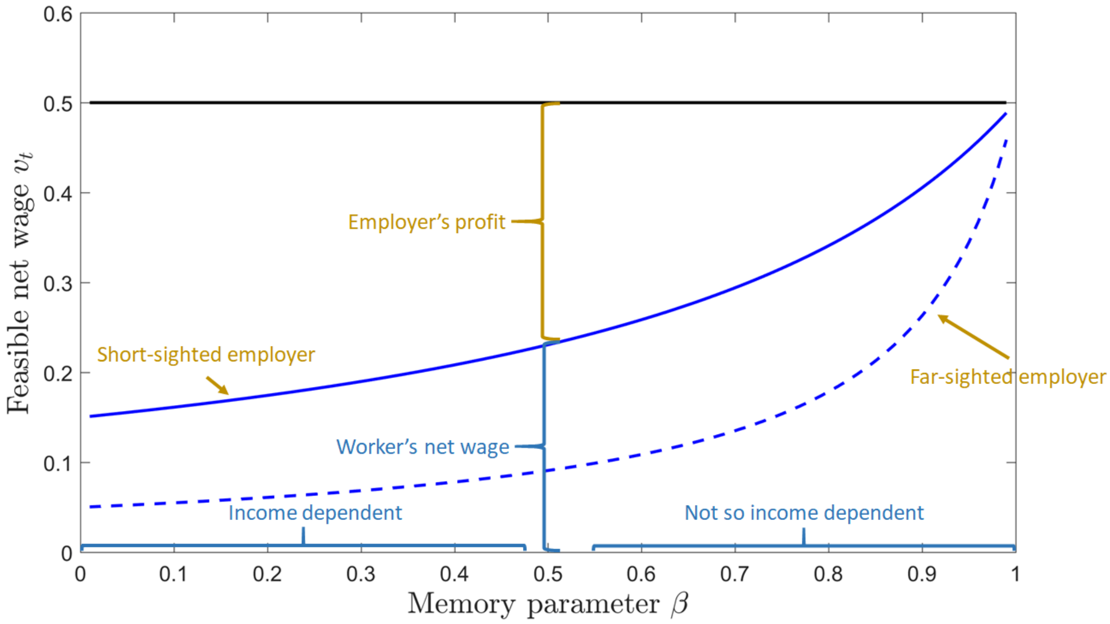

Figure 3: The feasible net wage (reference level $\bar{R}$) of the worker, and the profit of the employer ($V(\bar{R})$), as functions of the worker's memory decay parameter (β) at different employer discount factor values ($c = 1$, $\delta = 0.7$ short-sighted [blue line], $\delta = 0.9$ far-sighted [dashed blue line]).

almost as much as the profit in the current round. This means she can afford to stall production for a few rounds if workers have too high wage expectations. After the workers' reference decreases to levels she considers profitable, she will again offer acceptable contracts and can realize higher profit levels in the long run. If the employer has low δ, she concentrates on short-term profit. She is then forced to give higher wages to avoid missing rounds of production, even at the sacrifice of her profit level. Our result suggests the wage bargain between the worker and the employer is a waiting game: Whoever can afford to be more patient, will take a greater share from the fruits of production.

In this paper, we solved a deterministic benchmark solution for our proposed model that is stochastic in its most general form. Thereby we missed essential aspects of risk-awareness, and stochastically evolving worker preferences. We are planning to address these questions in the future.

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